Fall 2014

Source localization and tracking for possibly unknown signal propagation model

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SOURCE LOCALIZATION AND TRACKING FOR POSSIBLY UNKNOWN SIGNAL PROPAGATION MODEL

by

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A thesis submitted in partial fulfillment of the
requirements for the Master of Science
degree in Electrical and Computer Engineering
in the Graduate College of
The University of Iowa

December 2014

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CERTIFICATE OF APPROVAL

MASTER'S THESIS

This is to certify that the Master's thesis of

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has been approved by the Examining Committee for the thesis requirement for the Master of Science degree in Electrical and Computer Engineering at the December 2014 graduation.

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ACKNOWLEDGEMENTS

I would like to thank to my family and friends for always encouraging me to achieve my goal. I am very grateful and indebted to my advisor, Er-Wei Bai, for his scholarly insights, and fatherly enthusiasm.

PUBLIC ABSTRACT

This thesis discuses source localization and tracking under the condition when there is no much knowledge about the source and the background. A stationary and mobile source is to be localized and track using sensor upon receiving its signal by sensors. In reality the property of the signal from the source at a given environment may not be well known. Giving an assumption the signal received from the source is inversely proportional to the distance from the given sensor we are able to develop four algorithms.

In this thesis we first show how we can localize a stationary source using the signal received by fixed sensors. This is done upon comparing the signal received by pair of sensors. This was found to perform well even under the consideration of background noise. Localization of the stationary source is also done using a mobile sensor, in which we let a sensor move around the region of interest and use the signal to localize the position of the source.

A tracking of moving source is also discussed in which fixed sensors measured signals are used to localize the source position at each moment which constitute the trajectory of the moving source.

This thesis has an impact to the society in combating to the recently growing threat of terrorism. This way we would be able to localize and track the radioactive material before it makes major damage to the society.

ABSTRACT

This thesis considers source localization and tracking when both the signal propagation model and the source motion dynamics are unknown. Algorithms are developed for different scenarios. The algorithms are discussed when a source is stationary or mobile, under the condition when sensors are fixed or mobile. These algorithms exploit the strictly decreasing properties of the model in terms of distance, but do not depend on the form and the values of the models. Therefore, these algorithms could be applied when the signal propagation models and the source motion are unknown. The only assumption made is that the signal propagation strength decreases in distance. For a given performance specification, the optimal number and placement of the sensors is also discussed. Convergence and other properties of the algorithms are established under various noise assumptions.

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CHAPTER 1
INTRODUCTION

In this thesis we present some simple, effective algorithms in localizing and tracking a source without any information about its propagation model. The specific goal is to discover an effective, applicable and simple algorithm that can be applied to a number of different applications related to source localization and tracking, such as wireless communications, radioactive sources, and other applications. We assume that we do not have much information about the properties and dynamic motion of the source except the assumption that the propagation strength is inversely proportional to the distance of the source from the agent. This assumption is actually fairly reasonable. A number of models used in source localization and tracking in the litrature apply in some specific applications and are less effective in some other applications. Consequently the proposed algorithms are different from most previous works done in the area of source localization and tracking literature and can be applied to many applications without too much information on the signal propagation model and the motion dynamics. By contrast, the proposed algorithms are applicable and simulation supports the theoretical analysis.

1.1 Related works and our motivation

Over the last few years the problem of source localization and tracking has received tremendous attention in the literature [7, 16, 18, 21, 29] and there has been much effort in developing algorithms for such problems. The problem has an application in a wide range of areas, including wireless communication [8, 16, 20, 21, 23, 26], detection of radioactive materials [3, 10, 18, 19], airborne plume monitoring [11, 12, 25], mobile sensor networks [15, 16], as well as others. With all these important applications, numbers of algorithms have been proposed. For instance, smuggling of nuclear material poses a serious national security threat and therefore identification, localization and tracking of radioactive nuclear material have been extensively studied in other related literature [3, 19, 22, 29].

Some of the proposed algorithms in literature of source localizations and tracking are based on time difference of arrival or angle of arrival model [8, 26, 27]. These algorithms measure the time or angle difference between sensors data and exploit the cross correlation. These types of algorithms work in the context of wireless communication [4, 9] but may not be applicable in other applications, like radioactive material detection, since no time delay could be measured. In wireless communication we can have a formulation that comes from the time difference of arrival to the different agents, whereas in the literature of radioactive source localization we can only measure the number of photons emitted by the sources. In this case, it is difficult, may be even impossible, to measure the time difference of arrival at different sensors.

Other algorithms that have been used in the literature are based on the model of how the signal propagates [3, 28]. These approaches are useful in some applications but can be problematic in others, where having a reasonable model of how a signal propagates is not an easy task. In addition, even when a model is available, these models often have a number of unknown parameters that have to be estimated. As a result, such models are less effective or accurate. The most popular algorithm in the field is based on the received signal strength and the consequent triangulation or its variants. Suppose the exact relationship between the received signal strength and the distance between the unknown source and the sensor is available, then the distance between the source and the sensor can be uniquely determined based on the received signal strength. Equivalently, for a two dimensional problem, the unknown source lies on a circle with the sensor at its center with a known radius. With three or more noncollinear sensors, the unknown source location can be calculated. To improve the performance in the presence of noise, some robust algorithms are proposed [16] which are robust under various conditions.

To apply the signal-strength based algorithms, the relationship between the signal strength and the distance between the source and the sensor has to be available. This condition can be realistic in some applications but may not be realistic in others. For example, a path loss parameter characterizes the relationship between Received Signal Strength (RSS) measurements and distances and is notoriously uncertain, [24], and also could vary from as little as two to as high as eleven. A very common model in the literature for the received signal strength s at a given sensor position x_i [3, 19] is

$$s(x_i) = \frac{A^*}{d_i^{*2}} e^{-c^* d_i^*} \tag{1.1}$$

where $\phi^* = [A^*, c^*]^\top$ is a constant parameter vector that characterizes the signal strength, y^* is the unknown source location, $d_i^* = \|x_i - y^*\|$ is the distance between the sensor at x_i and y^*, A^* is the received signal strength at a unit distance from the source when $c^* = 0$, and c^* is the decaying rate due to, e.g., communication media or shielding material when the signal is Gamma ray count emanating from a radioactive source, [18, 19, 29]. Work in the litrature assumes that A^* and c^* are known or assumes c^* to be 0 [19]. So, the value of d_i^* can be uniquely determined from the value of $s(x_i)$, [7, 21].

How to localize and track the source y^* based on the received signal strength $s(x_i)$'s if A^* and c^* are unknown is an interesting and longstanding open question. In some applications, the relationship between the signal strength and the distance in between could be more complicated than (1.1) or only partially known, or completely unknown. For instance, in the presence of backscattering Gamma ray counts from radioactive materials, a model could be $\frac{A^*}{d_i^{*\alpha} d_i^{*2}} e^{-c^* d_i^*}$ for some unknown $\alpha > 0$ [17] which is obviously different from (1.1).

Localization from such uncertain signal models is an important, interesting but non-trivial problem. The results reported in this thesis address this question when there is only one source present and the received signal strength has a monotonal decreasing property in distance.

More formally, we assume that the received signal at the sensor location $x(t)$ at time t is

$$s(x(t)) = g(\|x(t) - y^*(t)\|) + e(t) = g(d^*(t)) + e(t) \tag{1.2}$$

where $x(t) \in R^n$ is the sensor location at time t, $y^*(t) \in R^n$ the unknown source location at time t, $d^*(t) = \|x(t) - y^*(t)\|$ the distance between $x(t)$ and $y^*(t)$, $e(t)$ the measurement noise at time t, and g is an *unknown* scalar function *strictly decreasing* in $\|x(t) - y^*(t)\|$. Further, let $y(t) \in R^n$ be an estimate of $y^*(t)$ at time t and $d(t) = \|x(t) - y(t)\|$ of $d^*(t)$. We consider both the settings where a single mobile sensor or multiple sensors make measurements located at time t_j at $x_i(t_j)$. Given a number of measurements $s(x_i(t_j))$ at $x_i(t_j)$, $i, j = 1, 2, ...,$ the goal is to find $y^*(t)$ from the measurements $s(x_i(t_j))$ and the sensor location $x_i(t_j)$.

1.2 Our contribution

The contribution of this thesis is to provide applicable algorithms for source localizations and tracking in cases where the source propagation model is not fully known, but it is monotonic in distance between the source and sensors. This has a number of applications as we discussed in the previous section.

1.3 Overview

The organization of this thesis is as follows. Three algorithms are developed along with their properties in Chapter two for a stationary source. Given the knowledge of a region where the source lies and a performance specification, this section also answers the question of how one should optimally place the sensors, and how many sensors should be used. Chapter three formulates robust tracking algorithms for a moving source. Simulations are presented in Chapter four. Chapter five discusses conclusion and future works.

CHAPTER 2
A STATIONARY SOURCE

In this chapter, we consider the case of a stationary source y^*. Hence, we will treat the source location y^* to be independent of time t. Both stationary and mobile sensors will be considered. Further, the following constitutes a standing assumption for this thesis.

Assumption 2.0.1. *The function $g(d)$ in (1.2) is a (possibly unknown) scalar function strictly decreasing in d, and is positive and twice differentiable for all $d > 0$. Further, the noise in (1.2) has finite variance, and is zero mean and independent from noise at other locations.*

2.1 Fixed location sensors

To illustrate ideas, first consider a noise free case. Since no relation is available between the signal strength and distance from the source, the localization cannot be performed by estimating the distance. However, in the absence of noise, the measurements $s(x_{i1})$, $s(x_{i2})$ from sensors at x_{i1} and x_{i2}, respectively, satisfy

$$s(x_{i1}) < s(x_{i2}) \Leftrightarrow d^*_{i1} > d^*_{i2} \Leftrightarrow \|x_{i1} - y^*\| > \|x_{i2} - y^*\|.$$

Indeed exploiting this and other inequalities is the core of this thesis. Let $l_i = x_{i1} - x_{i2}$ be the line segment connecting the points x_{i1} and x_{i2} and define

$$F_i = \{y \in R^n \mid s(x_{i1}) < s(x_{i2})\} \tag{2.1}$$

$$= \{y \in R^n \mid \|x_{i1} - y^*\| > \|x_{i2} - y^*\|\}.$$

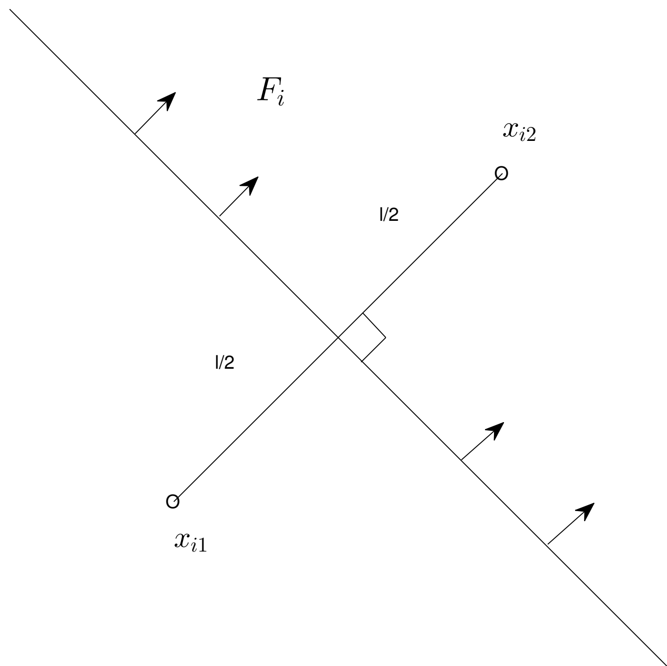

Figure 2.1: The set F_i.

Thus F_i is the half space of R^n defined by a hype-plane that (1) is equidistant from x_{i1} and x_{i2} and (2) has the line l_i as its normal direction as shown in Figure 2.1. (The lower half of the space if $s(x_{i1}) > s(x_{i2})$).

Consider now K pairs of measurements at location pairs x_{i1}, x_{i2} $i \in \{1, 2, ..., K\}$.

Define the feasible set F

$$F = \bigcap_{i=1}^{K} F_i = \bigcap_{i=1}^{K} \{y \in R^n \mid s(x_{i1}) < s(x_{i2})\} \tag{2.2}$$

Obviously the $y^* \in F$ and, in fact, F describes all possible y^*'s that are consistent with the data set, an idea reminiscent of the membership set [2]. We give a two dimensional example here. Let 4 measurements at x_i, $i = 1, 2, 3, 4$, satisfy

$$s(x_3) > s(x_2), s(x_2) > s(x_1), s(x_1) > s(x_4)$$

then, F_i, $i = 1, 2, 3$ and $F = \bigcap_{i=1}^{3} F_i$ can be constructed as shown in Figure 2.2 that describes all possible y^* consistent with the measurements.

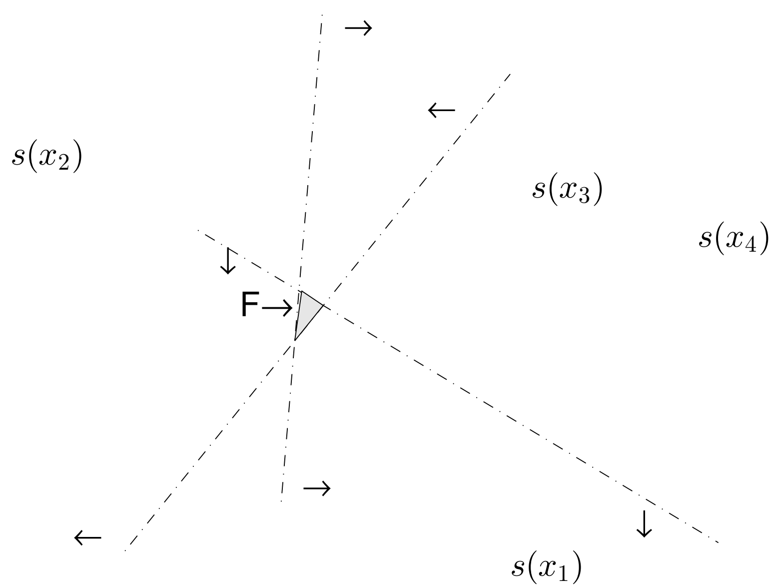

Figure 2.2: Illustration of the set F

Instead of finding a single estimate of y^*, this approach yields a set F that comprises all possible y^* consistent with the data. From a practical point of view, the localization accuracy is satisfactory if the "size" of F is small. The exact shape and size of F depends on many factors. In general, sufficiently rich sensor placement results in a small F and makes F converge to a singleton as $K \to \infty$ [1]. The condition will be discussed in the next section.

2.1.1 Minimal sensor number and optimal sensor placement

Suppose $I \in R^n$ is the region to be monitored. Without loss of generality, we assume that I is a bounded hyper-rectangle in R^n. Clearly, the feasible set F of (2.2) derived in the previous section is a convex polytope. The accuracy of localization can be quantified in terms of the volume or the diameter of F. We focus on volume here, noting that extension to diameters is trivial. Given the source y^*, and m sensors at $x_1, ..., x_m$, there are $K = m(m-1)/2$ pairs of measurements, $\{s(x_1), s(x_2)\}, ..., \{s(x_1), s(x_m)\}, ..., \{s(x_{m-1}), s(x_m)\}$. Suppose $F(y^*, x_1, ..., x_m)$ is the resulting feasible set. Define

$$V(y^*, x_1, ..., x_m) = \text{ Volume of } F(y^*, x_1, ..., x_m) \tag{2.3}$$

This volume measures the localization performance. The smaller the volume, better the localization. If there is no *a priori* knowledge of y^*, we may assume that y^* is uniformly distributed in I. Define the average performance

$$V_{ave}(x_1, ..., x_m) = \frac{1}{Vol(I)} \int_{y^* \in I} V(y^*, x_1, ..., x_m) dy^* \tag{2.4}$$

where $Vol(I)$ is the volume of I and $\frac{1}{Vol(I)}$ is the probability of y^* at any point in I. This is the average performance and is independent of the actual source location y^*. Thus, given m, the optimal placement of m sensors obeys

$$(x_1^*, ..., x_m^*) = arg \min_{x_1, ..., x_m} V_{ave}(x_1, ..., x_m). \tag{2.5}$$

Clearly, $V_{ave}(x_1^*, ..., x_m^*)$ is a non-increasing function of m. In fact, it goes to zero as $m \rightarrow \infty$.

Theorem 2.1.1. *Suppose the region of interest $I \in R^n$ is a bounded hyper-rectangle and $x_1^*, ..., x_m^*$ are as in (2.5). Then,*

$$\lim_{m\to\infty} V_{ave}(x_1^*, ..., x_m^*) = 0.$$

Proof. From the definition of $x_1^*, ..., x_m^*$, as $V_{ave}(x_1^*, ..., x_m^*)$ is nonincreasing, it suffices to show that there exists a sequence of m and a sequence of sensors $x_1, ..., x_m$ for which $V_{ave}(x_1, ..., x_m)$ defined in (2.4) goes to zero. To this end, partition I in each dimension into L equal parts so that I is the union of L^n boxes, whose volumes go to zero as L increases. Place sensors at each corner of the boxes. Then, $V_{ave}(x_1, ..., x_m)$ is proportional to the volume of each box and converges to zero as $L \to \infty$. Further, $V_{ave}(x_1^*, ..., x_m^*) \leq V_{ave}(x_1, ..., x_m)$. This completes the proof.

The sensor sequence used in the proof of Theorem 2.1.1 is of course suboptimal. To achieve effective sensor placement, we first pose the following problem whose solution may not be tractable. Then, we postulate a suboptimal though computationally tractable solution. Indeed, consider the problem: Given the region $I \in R^n$ and a performance specification $c_s > 0$, find the minimum number m of sensors and the corresponding optimal placement of sensors so that

$$V_{ave}(x_1^*, ..., x_m^*) \leq c_s \tag{2.6}$$

As posed, this is an intractable problem for several reasons. First, the calculation of (2.4) is non-trivial because an analytic expression of $V(y, x_1, ..., x_m)$ as a function of y is practically impossible even with a modest m. Secondly, the minimization of (2.5) is even harder. Note that V_{ave} is a non-linear, non-convex and possibly non-continuous function of x_i's and even with a modest m, it is unlikely to have an analytic expression of V_{ave} in terms of the x_i. Our approach in this thesis is not to find the global minimum, but a sub-optimal solution with a much lower computational complexity e.g., linear in m so that the problem of the "minimum" number of sensors and their placement satisfying the specified performance c_s can be solved quickly.

Before presenting the idea, note that though an analytic expression of $V_{ave}(x_1, ..., x_m)$ is impossible, it can be computed numerically by sampling: Select $y_1, ..., y_L$ as L uniformly distributed independent samples in I and calculate $V(y_i, x_1, ..., x_m)$, $i = 1, 2, ..., L$. These are themselves independent variables with bounded means and variances. Let

$$\bar{V}_{ave}^L(x_1, ..., x_m) = \frac{1}{L}\sum_{i=1}^{L} V(y_i, x_1, ..., x_m) \tag{2.7}$$

According to Law of Large Numbers for Random Functions, we have

$$\lim_{L\to\infty} \bar{V}_{ave}^L(x_1, ..., x_m) = V_{ave}(x_1, ..., x_m)$$

in probability. Our proposed suboptimal solution, with a much lower complexity, is based on the so called forward/backward selection, a well studied in the statistics literature [13]. Given a number m, the selection starts with two sensors at x_1 and x_2 that result in a F with its volume being $V_{ave}(x_1, x_2) = \frac{1}{2}Vol(I)$. Such a solution is optimal only if two sensors are given and easy to find, but needs not be unique. Then, one more sensor is added and its location is searched over I. More precisely, let $z_1, z_2, .., z_L$ be independent samples in I according to the uniform distribution. Let

$$x_3 = arg\min_{z_i} \bar{V}_{ave}^L(x_1, x_2, z_i) \tag{2.8}$$

for some L, where $\bar{V}^L_{ave}$ is defined in (2.7). Similar to (2.7) as $L \to \infty$, x_3 is a good estimate of

$$x^* = arg \min_x V_{ave}(x_1, x_2, x)$$

Fix the third sensor at x_3 and then the process is repeated for $x_4, x_5, ..., x_m$ one by one until all m sensors and their placement are determined. Once the forward selection is done in the order of $1, 2, ..., m$, the backward selection starts. First it frees x_1 and searches all $x \in I$ and replaces x_1 by x that minimizes $V_{ave}(x, x_2, ..., x_m)$. The process continues for $x_2, ..., x_m$ one by one until all sensors are checked.

Now we are in a position to state the forward/backward selection algorithm for the optimal placement of sensors, given the region I and the number m. Fix a large number $L > 0$.

Forward selection:

Step 1: Choose any two sensor locations x_1 and x_2 that provide a F with the average volume $V_{ave}(x_1, x_2) = 1/2Vol(I)$.

Step 2: Let the number of sensors chosen be l. Add one more sensor.

- Generate L sensor location samples $z_1, ..., z_L \in I$ uniformly and independently in I.
- For each z_i, generate L source samples $y_{i1}, ..., y_{iL}$ uniformly and independently in I.
- For each $z_i, y_{ij}, j = 1, 2, ..., L$, together with previously chosen $x_1, x_2, ..., x_l$, calculate

$$V(y_{ij}, x_1, ..., x_l, z_i), \text{ and}$$

$$\bar{V}_{ave}^{L}(x_1, ..., x_l, z_i) = \frac{1}{L}\sum_{j=1}^{L} V(y_{ij}, x_1, ..., x_l, z_i)$$

- Determine the optimal sensor placement

$$x_{l+1} = argmin_{z_i} \bar{V}_{ave}^{L}(x_1, ..., x_l, z_i)$$

Step 3: If $l + 1 = m$ stop and go to the backward selection. Otherwise, go back to Step 2.

Backward selection:

Step 1: Fix $x_2, x_3, ..., x_m$. Generate L sensor location z_i's uniformly and independently in I. For each z_i, generate L source location y_{ij}'s uniformly and independently in I. Calculate $\bar{V}_{ave}^{L}(z_i, x_2, ..., x_m) = \frac{1}{L}\sum_{j=1}^{L} V_{ave}(y_{ij}, z_i, x_2, ..., x_m)$ and find

$$x = arg \min_{z_i} \bar{V}_{ave}^{L}(z_i, x_2, ..., x_m)$$

Replace z_i by x and rename x by x_1.

Step 2: Continue the process for $x_2, x_3, ..., x_m$.

Now the algorithm for the optimal number and placement of the sensors for a given performance specification $c_s > 0$ can be summarized into one sentence. Applying the forward/backward selection algorithm for $m = 3, 4, ...$, the minimum number m and the corresponding sensor placement satisfying $\bar{V}_{ave}^{L}(x_1, x_2, ..., x_m) \le c_s$ solves the problem.

2.1.2 Minimizing noise effect

The previous results are derived assuming no noise. In the presence of noise

$$s(x_i(t_j)) = g(\|x_i(t_j) - y^*(t_j)\|) + e(t_j)$$

at time t_j, where $e(t_j)$'s are assumed to be independent and identically distributed random noise with zero mean and finite variance. The set F was defined based on $s(x_{i1}(t_j)) < s(x_{i2}(t_j))$ which may or may not be consistent with the actual inequality $g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)$. To guarantee, with a high probability, that $s(x_{i1}(t_j)) < s(x_{i2}(t_j))$ implies $g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)$, some averaging is needed to average out the effect of the noise. Since both the source and sensors are stationary, i.e., $y^*(t) = y^*$ and $x(t) = x$, a time average is an efficient way to go. Let $s(x_{i1}(t_j)), s(x_{i2}(t_j))$, $j = 1, 2, ..., L$ be L measurements from sensors at x_{i1} and x_{i2}. Define

$$\bar{s}(x) = \frac{1}{L}\sum_{j=1}^{L} s(x(t_j)) = g(\|x - y^*\|) + \frac{1}{L}\sum_{j=1}^{L} e(t_j)$$

This implies

$$\begin{aligned} \bar{s}(x_{i1}) - \bar{s}(x_{i2}) &= g(\|x_{i1} - y^*\|) - g(\|x_{i2} - y^*\|) \\ &+ \frac{1}{L}\sum_{j=1}^{L}(e_{i2}(t_j) - e_{i1}(t_j)) \end{aligned}$$

By Law of Large Numbers, the last term goes to zero as $L \to \infty$ with probability one. In other words, with a high probability for a large L, $\bar{s}(x_{i1}(t_j)) < \bar{s}(x_{i2}(t_j))$ implies $g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)$. In applications, the equation $s(x_{i1}) - s(x_{i2}) < 0$ in (2.1) should be replaced by

$$F_i = \{y \in R^n \mid \bar{s}(x_{i1}) < \bar{s}(x_{i2})\} \tag{2.9}$$

for some L.

The above result is asymptotic. For a finite L, the probability bound can also be calculated though conservative. Clearly given $\bar{s}(x_{i1}(t_j)) - \bar{s}(x_{i2}(t_j)) = -\epsilon < 0$, $g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)$ if

$$\left| \frac{1}{L} \sum_{j=1}^{L} (e_{i2}(t_j) - e_{i1}(t_j)) \right| \leq \epsilon$$

Let σ^2 be the variance of $e(t_j)$. By the Chebyshev inequality

$$Prob\,\{g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)\}$$

$$\geq Prob\left\{ \left| \frac{1}{L} \sum_{j=1}^{L} (e_{i2}(t_j) - e_{i1}(t_j)) \right| \leq \epsilon \right\} \geq 1 - \frac{2\sigma^2}{L\epsilon^2} \to 1$$

as $L \to \infty$. This provides a probability bound of confidence for a finite L.

2.2 A mobile sensor

We now consider source localization by a mobile sensor. The unknown source is still stationary. The problem is straightforward if the relation of the signal strength versus distance is available,which, however, is not assumed to be available in the thesis. Thus, the localization has to rely on other relations. Suppose there is a mobile sensor moving in a non-colinear, but otherwise arbitrary fashion as illustrated in Figure 2.3 for a two dimensional case. Again, if the mobile sensor makes m measurements at $x_1, ..., x_m$, it produces $m(m-1)/2$ pairs of data set, $\{s(x_1), s(x_2)\}, ..., \{s(x_1), s(x_m)\}, ..., \{s(x_{m-1}), s(x_m)\}$.

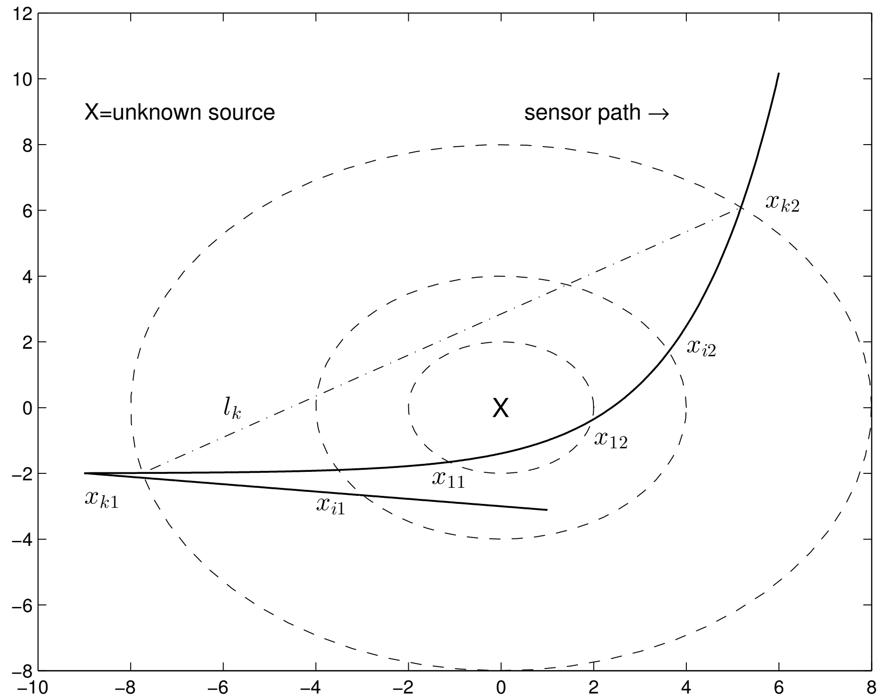

Figure 2.3: Sensor path.

2.2.1 Exploiting equality relation

Suppose along the sensor path, there are K pairs of points, $\{x_{i1}, x_{i2}\}_{i=1}^{K}$ at which $s(x_{i1}) = s(x_{i2})$. Then we have $d_i^* = \|x_{i1} - y^*\| = \|x_{i2} - y^*\|$ for an *unknown* d_i^*, $i = 1, ..., K$. Nonetheless, as

$$\|x_{i1} - y^*\|^2 = \|x_{i2} - y^*\|^2$$

this does imply that

$$(x_{i1} - x_{i2})^T y^* = \frac{1}{2}(\|x_{i1}\|^2 - \|x_{i2}\|^2)$$

or equivalently

$$\begin{bmatrix} (x_{1,1} - x_{1,2})^T \\ \vdots \\ (x_{K,1} - x_{K,2})^T \end{bmatrix} y^* = \frac{1}{2} \begin{bmatrix} \|x_{1,1}\|^2 - \|x_{1,2}\|^2 \\ \vdots \\ \|x_{K,1}\|^2 - \|x_{K,2}\|^2 \end{bmatrix} \tag{2.10}$$

Indeed, for this section we make an assumption here.

Assumption 2.2.1. *There exist $K \geq n$ and pairs of points $\{x_{i1}, x_{i2}\}_{i=1}^{K}$ on the path executed by the sensor, for which $s(x_{i1}) = s(x_{i2})$, $i = 1, ..., K$, and at least n of the vectors $x_{i1} - x_{i2}$ are linearly independent.*

This assumption is satisfied by generic trajectories of the mobile sensor. Clearly, under this assumption, the matrix on the left hand side of (2.10) has rank n and the solution of y^* is unique as summarized below.

Theorem 2.2.1. *Under Assumptions 2.0.1 and 2.2.1, the unknown source location y^* can be uniquely calculated by solving (2.10).*

Evidently, the knowledge of $g(\cdot)$ is immaterial and, beyond that, it satisfies the standing assumption.

2.2.2 Robustness

Small error analysis:

A key idea of the approach is that in the absence of noise

$$s(x_{i1}) = g(d_{i1}^*) = g(d_{i2}^*) = s(x_{i2}) \Rightarrow d_{i1}^* = d_{i2}^*$$

where $d_{ij}^* = \|x_{ij} - y^*\|,\ j = 1, 2$. The statement is however no longer true in the presence of noise. In the presence of noise, for $j \in \{1, 2\}$, $s(x_{ij})$ can be written as

$$s(x_{i,j}) = g(d_{i,j}^*) + v_{i,j} = g(d_{i,j}^* + \delta d_{i,j}),$$

for some unknown δd_{ij}. Therefore, instead of $d^*_{i1} = d^*_{i2}$, we have in the presence of noise that $s(x_{i1}) = s(x_{i2})$ implies

$$d^*_{i1} + \delta d_{i1} = d^*_{i2} + \delta d_{i2}$$

Now observe that under assumption 2.0.1, the unknown noise obeys:

$$v_{ij} = g(d^*_{ij} + \delta d_{ij}) - g(d^*_{ij}) = \frac{\partial g}{\partial d^*_{ij}} \delta d_{ij} + h.o.t.,\ j = 1, 2$$

where h.o.t. denotes high order terms. This implies

$$(d^*_{i1} + \delta d_{i1})^2 = (d^*_{i2} + \delta d_{i2})^2 \Rightarrow$$

$$\|x_{i1} - y^*\|^2 - \|x_{i2} - y^*\|^2 = d^{*2}_{i1} - d^{*2}_{i2} =$$

$$2 * d^*_{i2} \frac{1}{\frac{\partial g}{\partial d^*_{i2}}} v_{i2} - 2 * d^*_{i1} \frac{1}{\frac{\partial g}{\partial d^*_{i1}}} v_{i1} + h.o.t$$
$$= \beta_{i2} v_{i2} - \beta_{i1} v_{i1} + h.o.t.$$

for some bounded β_{i1} and β_{i2}. Let

$$e_i = \beta_{i2} v_{i2} - \beta_{i1} v_{i1},\ i = 1, 2, ..., K$$

which are mutually independent, zero mean, and have finite variance. In summary, in the presence of small noise, (2.10) becomes

$$\begin{bmatrix} (x_{1,1} - x_{1,2})^T \\ \vdots \\ (x_{K,1} - x_{K,2})^T \end{bmatrix} y^* = \frac{1}{2} \begin{bmatrix} \|x_{1,1}\|^2 - \|x_{1,2}\|^2 \\ \vdots \\ \|x_{K,1}\|^2 - \|x_{K,2}\|^2 \end{bmatrix} \tag{2.11}$$
$$+ \frac{1}{2} \begin{bmatrix} e_1 \\ \vdots \\ e_K \end{bmatrix} + h.o.t.$$

We then have the following Theorem whose proof is omitted as it follows from standard well known results.

Theorem 2.2.2. *Under Assumptions 2.0.1 and 2.2.1, suppose y is the least squares solution of (2.10). Then for sufficiently small noise variance, $y \to y^* + O(h.o.t.)$ in probability as $K \to \infty$.*

Weighted least squares:

Essentially, the analysis above is a small noise analysis so that higher order terms can be ignored. The translation is that the approach should work well if the noise is small, but the performance can not be guaranteed and deteriorates when the noise gets too large. To further improve performance, recall the Least Squares solution (2.11) minimizes

$$\min_y \sum_{i=1}^{K} ((x_{i1} - x_{i2})^T y - \frac{1}{2}(\|x_{i1}\|^2 - \|x_{i2}\|^2))^2$$

The error $(x_{i1} - x_{i2})^T y - \frac{1}{2}(\|x_{i1}\|^2 - \|x_{i2}\|^2)$ carries equal weight for each i. In many applications, while the noise has approximately the same energy level everywhere, the signal strength $s(x)$ declines with the distance between the sensor and the source. Suppose $g(\|x_{i1} - y^*\|) >> g(\|x_{j1} - y^*\|)$ but v_{i1} and v_{j1} have similar energy. $s(x_{i1}) = g(\|x_{i1} - y^*\|) + v_{i1}$ is much more reliable than $s(x_{j1}) = g(\|x_{j1} - y^*\|) + v_{j1}$ and so the former should carry much more weight than the later. Based on this observation, we consider the weighted least squares

$$\min_y \sum_{i=1}^{K} [w(i)((x_{i1} - x_{i2})^T y - \frac{1}{2}(\|x_{i1}\|^2 - \|x_{i2}\|^2))]^2 \tag{2.12}$$

The weight $w(i)$ can be chosen, e.g., as $\sqrt{s(x_{i1})}$ or $s(x_{i1})$.

Simulations presented later, confirm that while algorithms with $w(i) = 1, \sqrt{s(x_{i1})}, s(x_{i1})$, work well if the noise is small, for larger noise the weighted least squares algorithms perform substantially better than the unweighted one.

2.2.3 Exploiting inequality relation

The algorithm (2.10) is based on the idea that $s(x_{i1}) = s(x_{i2}) \Rightarrow d_{i1}^* = d_{i2}^*$ in the absence of noise. An advantage is that the exact relation $d_{i1}^* = d_{i2}^*$ can be exploited in developing the algorithm. Yet a switch from $s(x_{i1}) = s(x_{i2})$ to $s(x_{i1}) < s(x_{i2})$ or $s(x_{i1}) > s(x_{i2})$ also provides useful information, as was discused in the previous section.

Clearly in the absence of noise

$$s(x_{i1}) < s(x_{i2}) \Leftrightarrow d_{i1}^* > d_{i2}^* \Leftrightarrow \|x_{i1} - y^*\| > \|x_{i2} - y^*\|.$$

There are two important observations. First, the aim of the algorithm here is not to find a single estimate of y^* but to find the region or the set F that contains all possible y^* consistent with the data. From a practical point of view, the localization accuracy is satisfactory if the "size" of F is small. The exact shape and size of F depend on many factors. In general, a sufficient rich input sequence results in a small F and makes F converge to a singleton as $K \to \infty$ [1]. The exact sufficient rich condition will be discussed in the next section.

The second observation is that the sets F_i and F in (2.2), derived in the absence of noise, may not be valid even for a small noise. To make the algorithm robust with respect to noise, these sets have to be modified as

$$F_i = \{y \in R^n \mid s(x_{i2}) - s(x_{i1}) > \delta\} \tag{2.13}$$

$$F = \bigcap_{i=1}^{K} F_i = \bigcap_{i=1}^{K} \{y \in R^n \mid s(x_{i2}) - s(x_{i1}) > \delta\}, \tag{2.14}$$

for some $\delta > 0$. The slack variable $\delta > 0$ balances the robustness and efficiency of the algorithm. Note that

$$0 < s(x_{i2}) - s(x_{i1}) - \delta = g(d^*_{i2}) - g(d^*_{i1}) + v_{i2} - v_{i1} - \delta$$

implies that

$$d^*_{i1}) > d^*_{i2}), \text{ if } \delta > |v_{i1} - v_{i2}|. \tag{2.15}$$

Hence, a large δ makes the algorithm less efficient in terms of data usage but more robust because it tolerates a larger noise $\delta > |v_{i1} - v_{i2}|$. A small δ is just opposite. The choice of δ has to take *a priori* information about the noise into consideration.

Consider, for example an indepedent and identically distributed noise with support $(-\epsilon, \epsilon)$ for some $\epsilon > 0$. In such a case, let $\delta = 2\epsilon$. Then, the equation (2.15) is always true even in the presence of noise and the set F of (2.14) accurately describes all possible y^* that is consistent with the data even in the presence of noise. In summary, the algorithm discussed in this section that finds the set (2.14) is particularly effective for bounded noise.

2.2.4 Exploiting dynamics

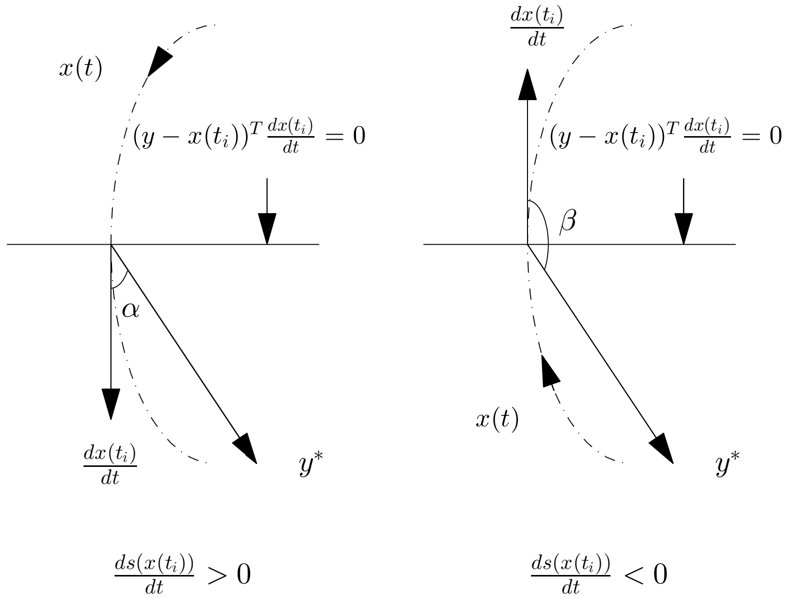

Figure 2.4: The set $(y - x_i)^T \frac{dx_i}{dt} = 0$.

In the previous sections, static properties of the unknown source y^* in relation to the sensor locations are investigated. In this section, we exploit the dynamic relationship of y^* in relation to the sensor location. Suppose a mobile sensor moves in R^n with the location $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T \in R^n$ at time t and continuously receives the signal $s(t) = s(x(t)) = g(d^*(t))$, where $d^*(t) = \|x(t) - y^*\|$. Suppose the derivative

$$s'(t) = \frac{ds}{dt} = \frac{ds}{dx_1}\frac{dx_1}{dt} + ... + \frac{ds}{dx_n}\frac{dx_n}{dt}$$

exists. Obviously $s'(t) > 0$ implies that the sensor is getting close to the source y^* and $s'(t) < 0$ the sensor moves away from the source y^*. Now suppose there is a t_i

such that $s'(t_i) = s'(x(t_i)) > 0$. From Figure 2.4, if $s'(t_i) > 0$ at $x(t_i)$, the angle α between $y^* - x(t_i)$ and

$$\left.\frac{dx}{dt}\right|_{t_i} = \left[\left.\frac{dx_1}{dt}\right|_{t_i}, \cdots, \left.\frac{dx_n}{dt}\right|_{t_i} \right]^\top$$

satisfies $|\alpha| < \pi/2$. Let $(y - x(t_i))^T dx/dt = 0$ be the hyper-plane in R^n which passes through $x(t_i)$ and take $\left.\frac{dx}{dt}\right|_{t_i}$ as its normal direction. Then, the unknown y^* has to be in the half space defined by

$$F_{+t_i} = \{y \in R^n \mid (y - x(t_i))^T \left.\frac{dx}{dt}\right|_{t_i} > 0\} \tag{2.16}$$

and further

$$y^* \in F_+ = \bigcap_i F_{+t_i} \tag{2.17}$$

Similarly, $s'(t_j) < 0$ implies $|\beta| > \pi/2$ as shown in Figure 2.4 and the unknown y^* must be in the half space defined by

$$F_{-t_j} = \{y \in R^n \mid (y - x(t_j))^T \left.\frac{dx}{dt}\right|_{t_j} < 0\} \tag{2.18}$$

and further

$$y^* \in F_- = \bigcap_j F_{-t_j} \tag{2.19}$$

In short, the unknown source location y^* must lie in the set

$$y^* \in F = F_+ \bigcap F_- \tag{2.20}$$

Just as the discussion of the feasible set in the previous section, the algorithm (2.20) does not give rise to a single estimate of y^* but does provide a set that captures all possible y^*. The set F is referred to as the feasible set again and this algorithm can be more powerful than algorithms providing a single estimate.

To illustrate, we give a two dimensional example as shown in Figure 2.5. The area of interest is rectangle. The sensor path consists of 2 line segments in parallel to the horizontal and vertical axis respectively. In the figure, let $x(t_1)$ and $x(t_2)$ be two points on the sensor path so that $y^* - x(t_1)$ and $y^* - x(t_2)$ are perpendicular to the sensor path. Clearly, $s'(x(t_i - \delta t)) > 0$ and $s'(x(t_i + \delta t)) < 0$, $i = 1, 2$, for small enough $\delta t > 0$. From the discussion above, the unknown y^* is on the left hand side of $F_{+(t_1 - \delta t)}$ as in (2.16) and the right hand side of $F_{-(t_1+\delta t)}$ as in (2.18). These two lines can be made arbitrarily close by letting $\delta t \to 0$. Similarly, y^* must be sandwiched between $F_{-(t_2+\delta t)}$ and $F_{+(t_2-\delta t)}$, and again $F_{-(t_2+\delta t)}$ and $F_{+(t_2-\delta t)}$ can be made arbitrarily close by letting $\delta t \to 0$. Therefore, by letting $\delta t \to 0$,

$$y^* \in F_{+(t_1-\delta t)} \cap F_{+(t_2-\delta t)} \cap F_{-(t_1+\delta t)} \cap F_{-(t_2+\delta t)} \to \{y^*\}$$

or the feasible set contains only one point y^*. The idea can be easily extended to any dimensional space R^n.

Theorem 2.2.3. *Let the region of interest I be a hyper-rectangle in R^n whose axes are parallel to the coordinate axes respectively and let the sensor path be n line segments l_k, $k = 1, 2, ..., n$. Each line l_k is parallel to the kth axis. Let $x(t_k)$ be a point on the line l_k so that $y^* - x(t_k)$ is perpendicular to the line l_k (such an $x(t_k)$ always exists). Construct F_{+t_k} of (2.16) and F_{-t_k} of (2.18) at $x(t_k - \delta t)$ and $x(t_k + \delta t)$ respectively for small enough $\delta t > 0$. Then, $y^* \in F \in \bigcap_{k=1}^{n}(F_{+(t_k-\delta t)} \cap F_{-(t_k+\delta t)})$.*

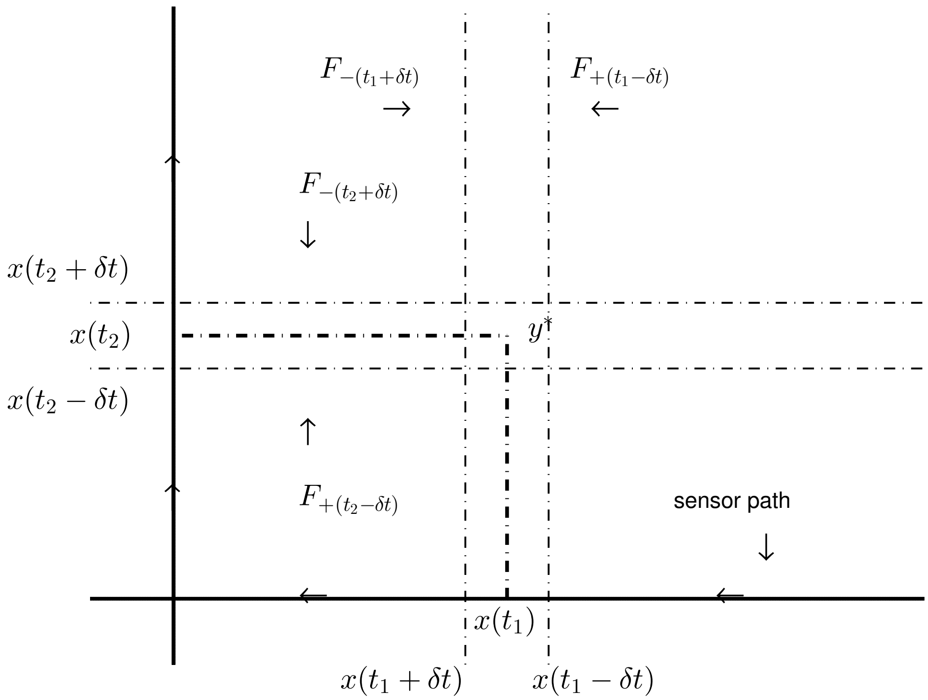

Figure 2.5: Illustration of F.

Further let $\delta t \to 0$,

$$y^* \in F \to \{y^*\}.$$

Proof: Clearly, $y^* \in \bigcap(F_{+(t_k-\delta t)} \cap F_{-(t_k+\delta t)})$ that is hyper-rectangle with the diagonal length

$$\sqrt{\sum_{k=1}^{n}(x(t_k+\delta t)-x(t_k-\delta t))^2}$$

which converges to zero as $\delta t \to 0$. This completes the proof.

In fact having n lines in the sensor path is not necessary. What is important is that $s(t)$ have n local maxima. Suppose $s(t_1)$ is a local maximum. Then $s'(t_1-\delta t) > 0$ and $s'(t_1+\delta t) < 0$ for small enough $\delta t > 0$. The corresponding half spaces $F_{+(t_1-\delta t)}$ and $F_{-(t_1+\delta t)}$ can be constructed at $x(t_1-\delta t)$ and $x(t_1+\delta t)$ respectively, and $y^* \in$

$F_{+(t_1-\delta t)} \bigcap F_{-(t_1+\delta t)}$. Further, the two planes $(y-x)^T dx/dt$ at $t_1 \pm \delta t$ that define the half spaces are almost parallel and converge to each other as $\delta t \to 0$. Based on this observation, the following result can be established.

Theorem 2.2.4. *Let the region of interest I be in R^n and $x(t)$ be the sensor path. Suppose $s(t)$ achieves n local maxima at $x(t_1), x(t_2), ..., x(t_n)$. Construct $F_{+(t_k-\delta t)}$ of (2.16) and $F_{-(t_k+\delta t)}$ of (2.18) at $x(t_k - \delta t)$ and $x(t_k + \delta t)$ respectively for small enough $\delta t > 0$. Then, $y^* \in F \in \bigcap_{k=1}^{n}(F_{+(t_k-\delta t)} \cap F_{-(t_k+\delta t)})$. Further, let $\frac{dx}{dt}\big|_{t_1}, ..., \frac{dx}{dt}\big|_{t_n}$ be linearly independent. Then, as $\delta t \to 0$,*

$$y^* \in F \in \bigcap_{k=1}^{n}(F_{+(t_k-\delta t)} \cap F_{-(t_k+\delta t)}) \to \{y^*\}.$$

From the above observations we make a few comments.

- Not all samples of $s(t)$ or $s'(t)$ are equally important in establishing the feasible set F. The crucial roles are played by the local maxima of $s(t)$.
- The derivation of the feasible set F is in the continuous time. In practice, only sampled data is available. To this end, let $\Delta t > 0$ be the sampling interval and $s(k\Delta t)$ the measurements. What we are looking for are the local maxima of $s(k\Delta t)$, the times $k\Delta t$ and the sensor location $x(k\Delta t)$.
- To average out the effect of noise, a smoothing version of s can be applied, e.g. $s(k\Delta t) = \frac{1}{2M+1}\sum_{i=k-M}^{k+M} s(i\Delta t)$. M balances the sensitivity to noise and the accuracy of the algorithm. Note the weights do not have to be equal. In fact numerical differentiation from noisy sampled data is an extensively studied topic and a number of well developed algorithms are available in the literature,

e.g, the Lanczos differentiator, the Savitzky-Golay filter and orthogonal function projection [14] all of which can be applied here.

- Suppose $s(k_1\Delta t)$ is a local maximum. The corresponding $F_{+(k_1-m)\Delta t}$ and $F_{-(t_k+m)\Delta t}$ can be constructed at $x((k_1-m)\Delta t)$ and $x((k_1+m)\Delta t)$ respectively. A small m makes F small but may miss the true but unknown value y^* if the noise is large.

CHAPTER 3
A MOVING SOURCE

In this chapter, we consider tracking a moving source using stationary sensors, without explicit knowledge of the source dynamics. Localization of the source $y^*(t_j)$ at time t_j is completely based on the measurements obtained at t_j. If *a priori* knowledge on the movement of the source is available, it can be incorporated into the localization and tracking algorithm. However in reality, any assumption on the unknown source movement is unrealistic, particularly for a non-cooperating source.

Since no motion model is assumed and $y^*(t_j)$ is completely characterized by the information at t_j, the tracking problem of $y^*(t_j)$ is actually obtaining a sequence of location estimates of $y^*(t)$ at $t = t_1, t_2, ...$ Thus, the method presented in chapter two can be applied as summarized below.

Tracking algorithm:

Step 1: Given the performance specification $c_s > 0$, apply the algorithm for the optimal number and placement of the sensors in chapter two to find the minimum number m and the corresponding sensor placement satisfying the performance specification.

Step 2: At each t_j, collect measurement $s(x_i(t_j))$, $i = 1, 2, ..., m$ and construct $F(t_j)$ as in equation (2.2). $F(t_j)$ characterizes the unknown source $y^*(t_j)$ location at time t_j.

Though, the tracking algorithm is similar to the case for a stationary source and sensors, the effect of noise is completely different. In chapter two, the source is stationary and time averaging works well. For a moving source, time averaging is complicated as both $y^*(t)$ and $s(x(t)) = g(\|x - y^*(t)\|) + e(t)$ are time varying. We discuss the ways to deal with noise for a moving source in the following two sub sections.

3.1 Threshold approach

The problem with noise is again that $s(x_{i1}(t_j)) < s(x_{i2}(t_j))$ does not necessarily imply $g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)$. Thus, the resulting $F_i(t_j)$ based on $s(x_{i1}(t_j)) < s(x_{i2}(t_j))$ could provide completely false conclusion on the unknown source location $y^*(t_j)$. To make the algorithm robust, the definition of $F_i(t_j)$ is modified as

$$F_i(t_j) = \{y \in R^n \mid s(x_{i1}(t_j) - s(x_{i2}(t_j)) \leq -c < 0\} \tag{3.1}$$

for some $c > 0$ and

$$F(t_j) = \bigcap F_i(t_j) \tag{3.2}$$

The addition of $c > 0$ robustifies the algorithm. The question is how to choose c? The answer depends on the level of confidence in $g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)$ given $s(x_{i1}(t_j)) < s(x_{i2}(t_j))$. As the sensors are stationary, $s(x_{i1}(t_j)) - s(x_{i2}(t_j)) \leq -c$ implies $g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)$ if and only if $-c + e_{i1}(t_j) - e_{i2}(t_j) < 0$. We consider two cases.

1. Bounded noise: Suppose $e(t)$ is unknown but bounded, i.e., $|e(t)| < \epsilon$ for all t. Then let $c = 2\epsilon$ which implies $-c + e_{i1}(t_j) - e_{i2}(t_j) < 0$. Simply put, $s(x_{i1}(t_j) - s(x_{i2}(t_j)) \leq -c$ guarantees $g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)$ or $F_i(t_j)$ is always correct by such a choice.

2. Random noise: For simplicity, we assume that the noise in the rest of this section is independent and identically distributed Gaussian with zero mean and finite variance σ^2. Then $e_{i1}(t_j) - e_{i2}(t_j)$ is also independent and identically distributed Gaussian of zero mean and variance $2\sigma^2$. Given $s(x_{i1}(t_j)) - s(x_{i2}(t_j)) \leq -c$, the probability

$$Prob\{g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)\}$$

$$= Prob\{-c + e_{i1}(t_j) - e_{i2}(t_j) < 0\} \geq \begin{cases} 0.978 & if\ c = 2\sqrt{2}\sigma \\ 0.991 & if\ c = 3\sqrt{2}\sigma \\ \to 1 & if\ c \to \infty \end{cases}$$

Thus with $c = 3\sqrt{2}\sigma$, $s(x_{i1}(t_j)) - s(x_{i2}(t_j)) \leq -c$ implies $g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)$, i.e. $F(t_j)$ is correct, with a high level of confidence.

From the above analysis, a large c makes the algorithm very robust. The price paid is that all the measurements that do not satisfy $s(x_{i1}(t_j)) - s(x_{i2}(t_j)) \leq -c$ are discarded. Thus, c balances the robustness of the algorithm with efficient data use. Another critical factor is the localization accuracy or the size of $F(t_j)$. When the number m of the sensors increases, the size of $F(t_j)$ decreases. With an increasing c, the consistency between $s(x_{i1}(t_j)) - s(x_{i2}(t_j)) \leq -c$ and $g(\|x_{i1}(t_j) - y^*(t_j)\|) < g(\|x_{i2}(t_j) - y^*(t_j)\|)$ gets larger and at the same time, the number of constraints that satisfy $s(x_{i1}(t_j)) - s(x_{i2}(t_j)) \leq -c$ decreases that lead to a large size $F(t_j)$. The conclusion is that to make the algorithm robust and maintain the same accuracy, the number of sensors has to increase while c increases.

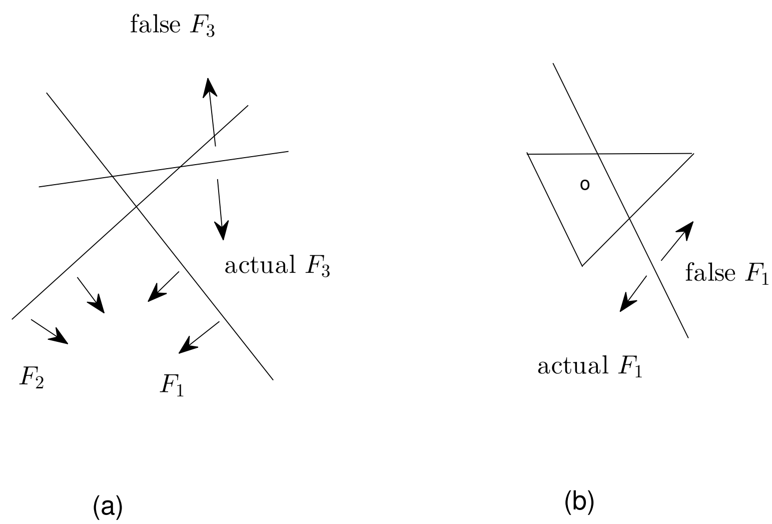

Figure 3.1: Violated Constraints

3.2 Majority voting and a few violated constraints

In some applications, the number of the sensors may not be large. In such a scenario, a large threshold c is not possible and c has to be small. The question is what can one do to reduce the effect of noise? With large noise and just a few sensors, the options are limited. Consider instead the case where the noise is small. In this case:

- Most of the constraints remain valid and only a small number of constraints are violated, though which precise ones are violated is unknown. This suggests the use of majority voting.
- Further, by deleting violated constraints, the resulting $F(t_j)$ constructed by

the remaining constraints is acceptable as long as there are just a few violated constraints.

Then, the effect of noise can be reduced by the idea of majority voting and a few violated constraints [1] by identifying and removing these violated constraints. Note that there are two types of violated constraints. The first occurs when for some i, k $F_i(t_j) \bigcap F_k(t_j)$ or equivalently $F(t_j)$ is empty. Consider Figure 3.1a where the true $F(t_j) = \bigcap_{i=1}^{3} F_i(t_j)$ exists. Because of noise, $F_3(t_j)$ is falsely constructed that makes $F(t_j)$ empty.

The second violation is more subtle. Let us say $F_1(t_j)$ is falsely constructed because of noise as in Figure 3.1b, where "o" is the unknown source $y^*(t_j)$ and the polytope is $\bigcap_{i=2}^{K} F_i(t_j)$ without $F_1(t_j)$. Obviously, F_1 would be known to be false if $y^*(t_j)$ were available. However, $y^*(t_j)$ is not available and so there is no way of knowing if F_1 is correct or false. However under the assumption that $\bigcap_{i=2}^{K} F_i(t_j)$ is small and acceptable, the intersection of F_1 and $\bigcap_{i=2}^{K} F_i(t_j)$ is even smaller and not far from $y^*(t_j)$, thus making the localization result not exact but acceptable.

Now the question is how to detect if only a small number of constraints are violated and further to identify such violated constraints. Note that $F(t_j) = \bigcap F_i(t_j)$ and each $F_i(t_j)$ is a half space defined by a hyper-plane which is in the form of $a_i^T x \leq b_i$ for some $a_i \in R^n$ and $b_i \in R$. If some constraints are violated, the equation $Ax \leq b$ does not have any solution for x, where $A = \begin{bmatrix} a_i^T \\ \vdots \\ a_K^T \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ \vdots \\ b_K \end{bmatrix}$. To find the violated constraints, one may consider

$$\begin{gathered}\min \|p\|_2 \\ \textit{subject to } Ax \leq b + p,\ \ p \geq 0\end{gathered}$$

where $p \in R^n$ as suggested in [5]. If one removes the constraint $a_i^T x \leq b_i$ if $p_i > 0$, the remaining equation $Ax \leq b$ still admits a solution. Thus the remaining constraints are consistent and $F(t_j)$ is non-empty. Recall our goal is to find and remove the minimum number of violated constraints that results in a non-empty $F(t_j)$. The vast compressive sensing literature [6] recommends the use of the 1-norm rather than the 2-norm. In summary, the robust tracking algorithm for a small noise can be stated as follows:

1. At each t_j, collect $s(x_{i1}(t_j)) - s(x_{i2}(t_j)) \leq -c < 0$ for some small $c > 0$ and the corresponding constraint $a_i^T x \leq b_i$ that define the half space $F_i(t_j)$.

2. Solve the optimization problem,

$$\begin{gathered}\min \|p\|_1 \\ \textit{subject to } Ax \leq b + p,\ \ p \geq 0\end{gathered} \tag{3.3}$$

3. Remove $a_i^T x \leq b$ from the constraint or equivalently discard $F_i(t_j)$ if $p_i > 0$ and construct $F(t_j)$ from the remaining F_i's.

4. $F(t_j)$, $j = 1, 2, ...$ characterizes the trajectory of the unknown $y^*(t_j)$.

CHAPTER 4
NUMERICAL EXAMPLES

We simulated a numerical example with the received signal strength

$$s(x(t)) = g(\|x(t) - y^*(t)\|) + e(t)$$
$$= \frac{100}{\|x(t) - y^*(t)\|^2} e^{-0.01\|x(t)-y^*(t)\|} + e(t) \tag{4.1}$$

No information of g was used in the simulation.

4.1 Numerical examples for stationary source

First considered was a case that the unknown source was fixed at $y^* = [0, 0]^\top$ and the sensor locations were also stationary. The region I to be considered was 30 units long and 30 units wide or with the area of 900 unit squares. Suppose the performance specification was $\bar{V}^L_{ave}(x^*_1, ..., x^*_m) \leq 1$ (unit squares) with $L = 50$. By applying the optimal number and placement of the sensors in chapter two for each m as shown in Figure 4.1, it was shown the optimal performance of $\bar{V}^L_{ave}(x^*_1, ..., x^*_{14}) =$ 1.28 (unit squares) and the optimal performance of $\bar{V}^L_{ave}(x^*_1, ..., x^*_{15}) = 0.6212$ (unit squares). Thus, we determined the placement of 15 sensors, using the algorithm in (2.9) of Chapter 2 with $L = 10$.

To this end we set $e(t) = 0.1\eta$ with $\eta \sim N(0, 1)$ and is independent and identically distributed. The algorithm resulted in $F = \bigcap F_i$ shown in Figure 4.2 with the area 0.0570 (unit squares) where "O" is the unknown location of y^*.

We then considered a mobile sensor whose path was described in Figure 2.3. More precisely, the path had two pieces. The first piece, whch is a function of time,

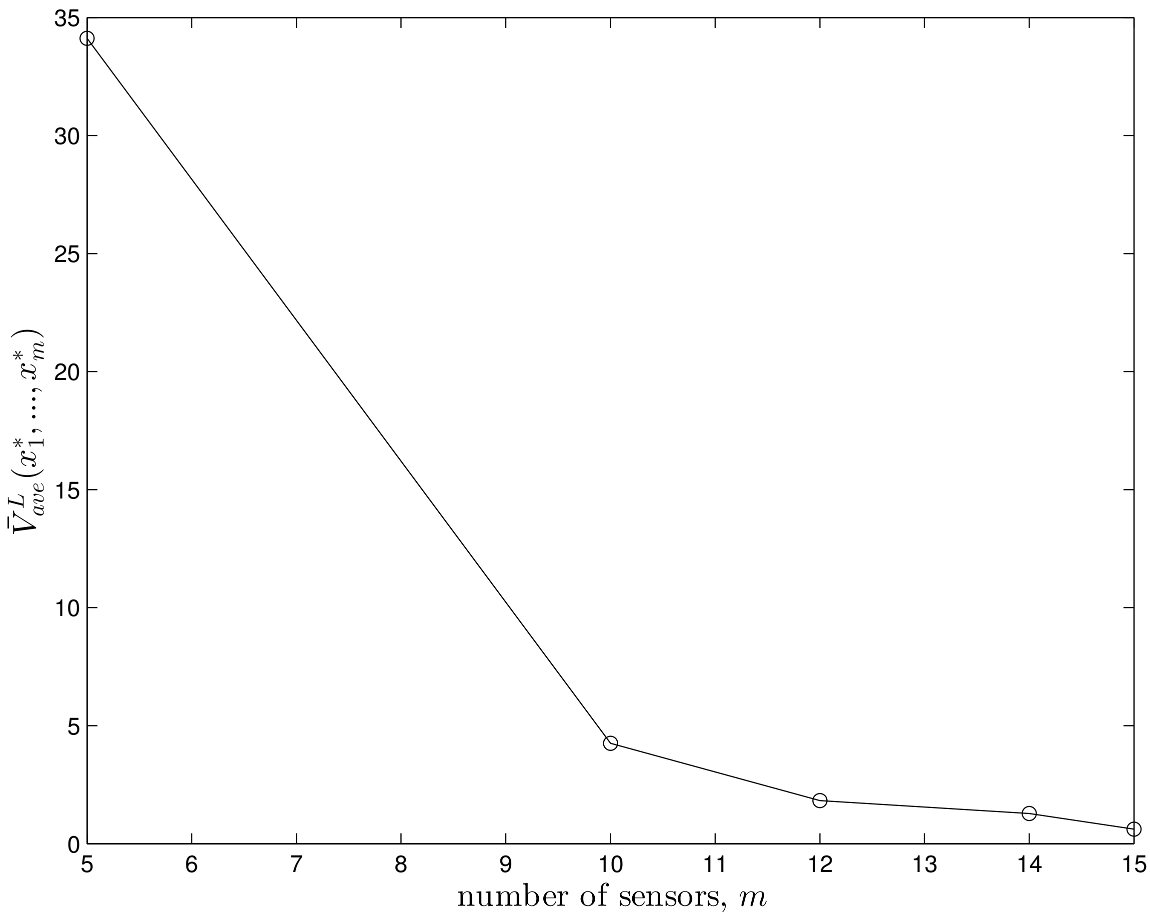

Figure 4.1: Optimum performance of $\bar{V}^L_{ave}(x^*_1, ..., x^*_m)$ to different number of sensors

at time t was $x(t) = [t, -0.1455t - 3]^\top$ for $t \in [-7, 1]$ and the second one $x(t) = [t, \exp(0.5(t-1)) - 2]^\top$ for $t \in [1, 5]$. In simulation, only the sampled $s(i\Delta t) = s(x(i\Delta t))$'s were available for $\Delta t = 0.02$. The noise $e = \alpha\eta$ and α adjusted the noise contribution.

The least squares algorithms (2.12) with various weights were applied. Along the path, 40 pairs of x_{i1}, x_{i2}'s were chosen so that

$$s(x_{i1}) = g(\|x_{i1} - y^*\|) + e_{i1} = g(\|x_{i2} - y^*\|) + e_{i2} = s(x_{i2})$$

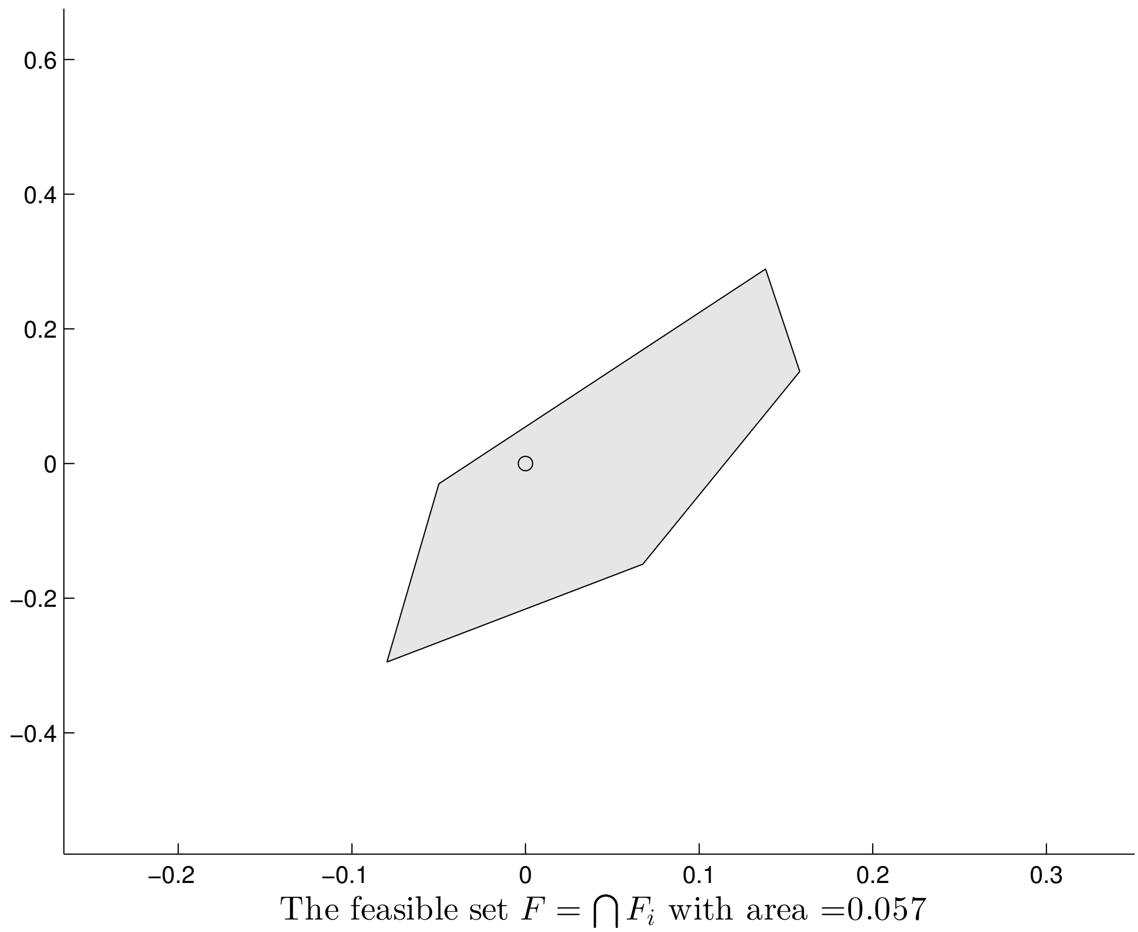

Figure 4.2: The feasible set F.

Figure 4.3 which are the averages of 50 Monte Carlo runs and Table 1 show the estimation errors $\|y - y^*\|^2$ for the weights $w(i) = 1$, $w(i) = \sqrt{s(x_{i1})}$ and $w(i) = s(x_{i1})$ respectively for different α's. The simulation results confirmed the theory, i.e. all three algorithms work well if the noise is small ($\alpha \leq 0.4$) and the weighted least squares algorithms perform substantially better when the noise is not small ($\alpha \geq 0.6$). In fact considering that the search was in a rectangle of 30 by 30, the algorithms with all three weights performed reasonably well for all $\alpha \leq 1$.

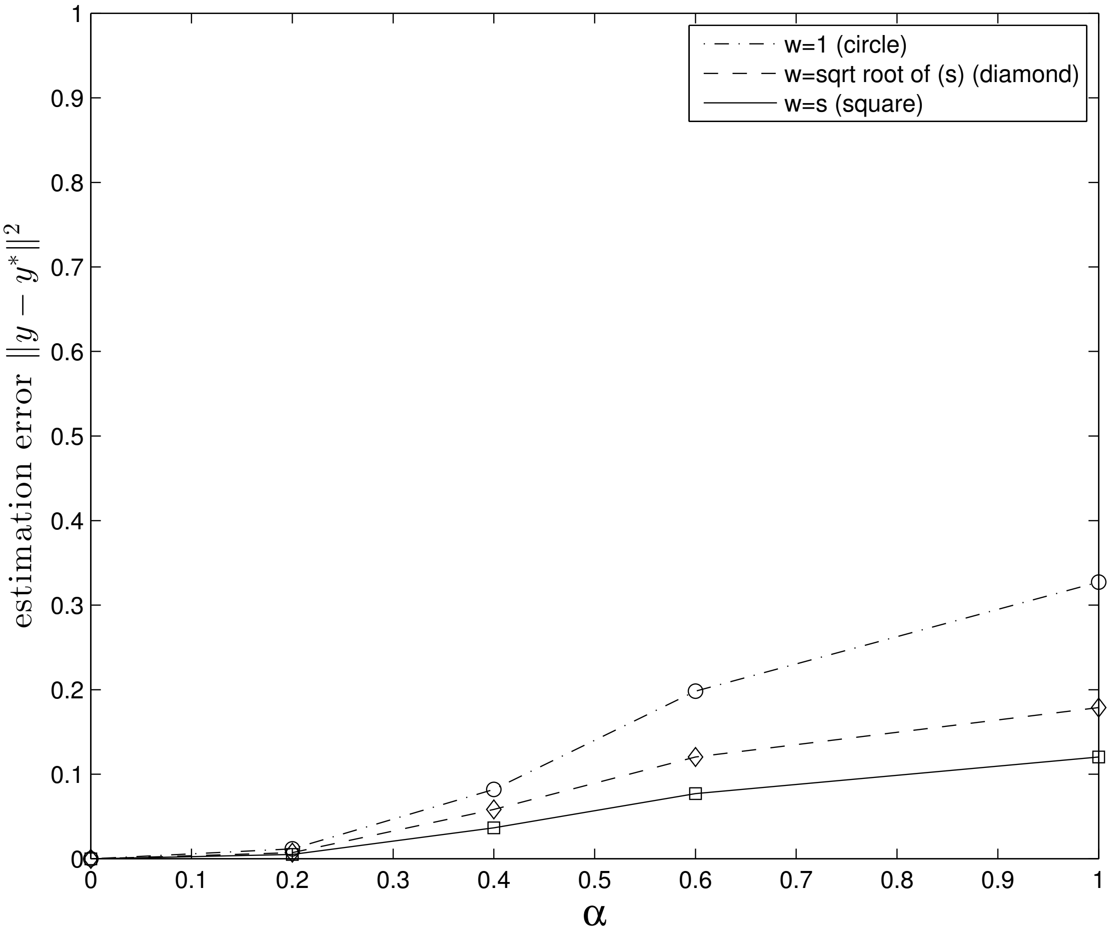

Figure 4.3: Estimation errors for various α.

The feasible set approach (2.14) was simulated. The sensor path is the same as before. Only four sets of measurements along the path were used at

$$\{\begin{pmatrix}-0.8020\\-1.5938\end{pmatrix}, \begin{pmatrix}1.600\\-0.6501\end{pmatrix}\}, \{\begin{pmatrix}-2.6020\\-1.8349\end{pmatrix}, \begin{pmatrix}0.1\\-3.015\end{pmatrix}\},$$

$$\{\begin{pmatrix}2.7980\\0.4541\end{pmatrix}, \begin{pmatrix}-2.0020\\-1.7771\end{pmatrix}\}, \{\begin{pmatrix}2.7980\\0.4541\end{pmatrix}, \begin{pmatrix}0\\-3\end{pmatrix}\}$$

as shown in Figure 4.4 as d, x, o, s respectively. For simulation, noise was assumed to be independent and identically distributed uniform in $(-0.5, 0.5)$ and the slack variable $\delta = 2 * 0.5 = 1$. Figure 4.4 shows F_i, $i = 1, 2, 3, 4$, which is a half space represented by a line segment in the figure, and the feasible set $F = \bigcap F_i$ which provides all possible y^* consistent with the data.

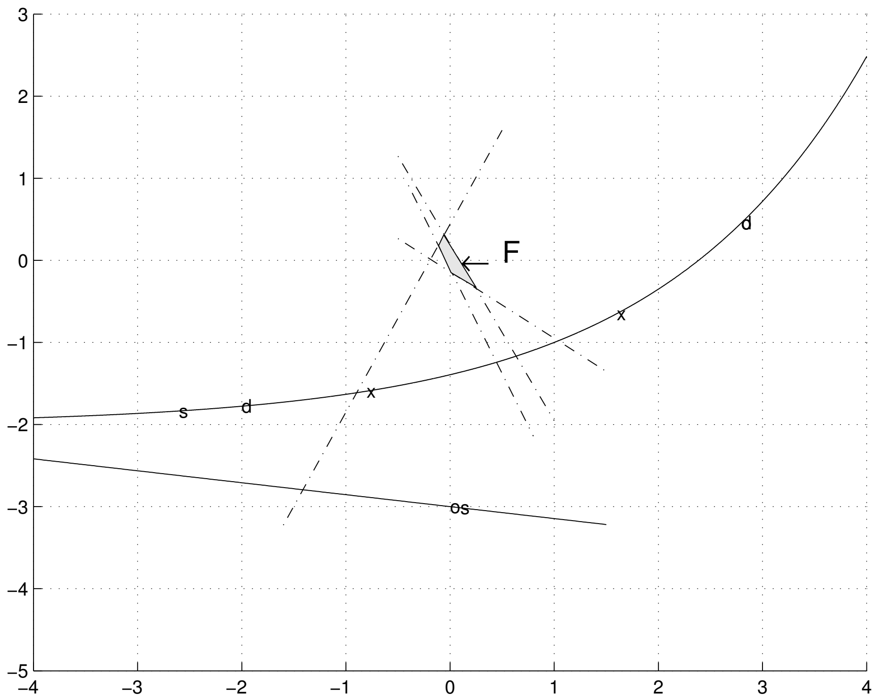

Figure 4.4: The feasible set $F = \bigcap F_i$.

The algorithm (2.20) exploiting dynamics was also implemented with $M = 30$ and $l = 5$ to smooth out $s(k\Delta t)$'s as discussed in the previous chapter. The noise was independent and identically distributed Gaussian of $N(0, 4)$. Two local maxima of $s(i\Delta t)$ were found at $t_1 = 73\Delta t = 1.46$ and $t_2 = 777\Delta t = 15.54$. Then, $F_{+(t_1 - l\Delta t)}$, $F_{-(t_1 + l\Delta t)}$, $F_{+(t_2 - l\Delta t)}$ and $F_{-(t_1 + l\Delta t)}$ were constructed. From Figure 4.5, the true but

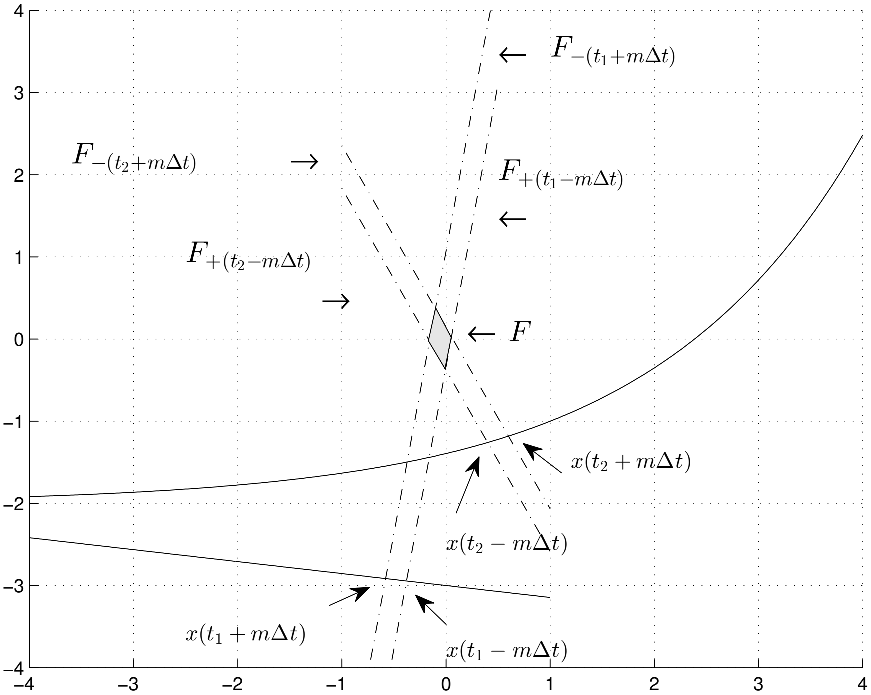

Figure 4.5: The feasible set $F = \bigcap F_i$.

unknown y^*

$$y^* \in F \in F_{+(t_1-l\Delta t)} \cap F_{-(t_1+l\Delta t)} \cap F_{+(t_2-l\Delta t)} \cap F_{-(t_1+l\Delta t)}$$

and F could be made further small by reducing l.

4.2 Numerical examples for moving source

Finally, tracking of a moving source was considered. The unknown source path was a random Brownian motion $y^*(t_j)$, $j = 1, 2, .., 20$ in I. Three cases were simulated, no noise, small noise and significant noise. Figure 4.6 shows the tracking results $F(t_j)$, $j = 1, 2, ..., 20$ with no noise by applying the Tracking algorithm in chapter three with 15 sensors, where "O"'s are the true but unknown $y^*(t_j)$ at t_j and

	$\alpha = 0.6$	$\alpha = 1$
$w(i) = 1$	$(0.2050, -0.4963)^T$	$(-0.5992, 0.0631)^T$
$w(i) = \sqrt{s(x_{i1})}$	$(0.1452, -0.3692)^T$	$(0.2311, -0.5568)^T$
$w(i) = s(x_{i1})$	$(0.0372, -0.0936)^T$	$(0.0566, -0.1735)^T$

Table 4.1: Estimates for different weights.

the arrows indicates the direction of source motion. The gray areas are $F(t_j)$'s.

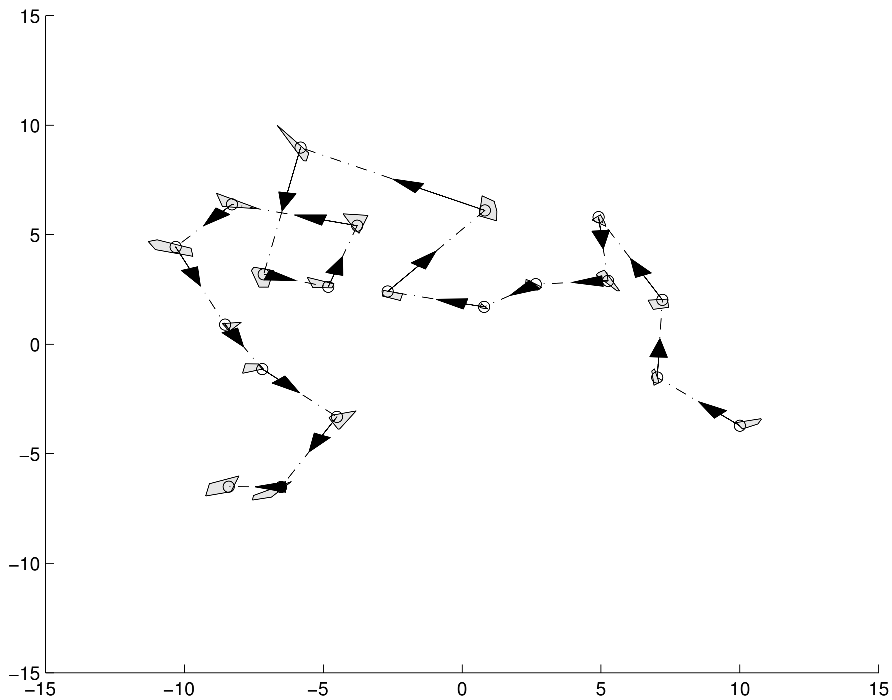

Figure 4.6: The feasible sets for a moving source

We then added a small noise 0.1η. Since only very few constraints were violated determined by (3.3), the algorithm of a few violated constraints developed in chapter three was applied. The result shown in Figure 4.7 is surprisingly good.

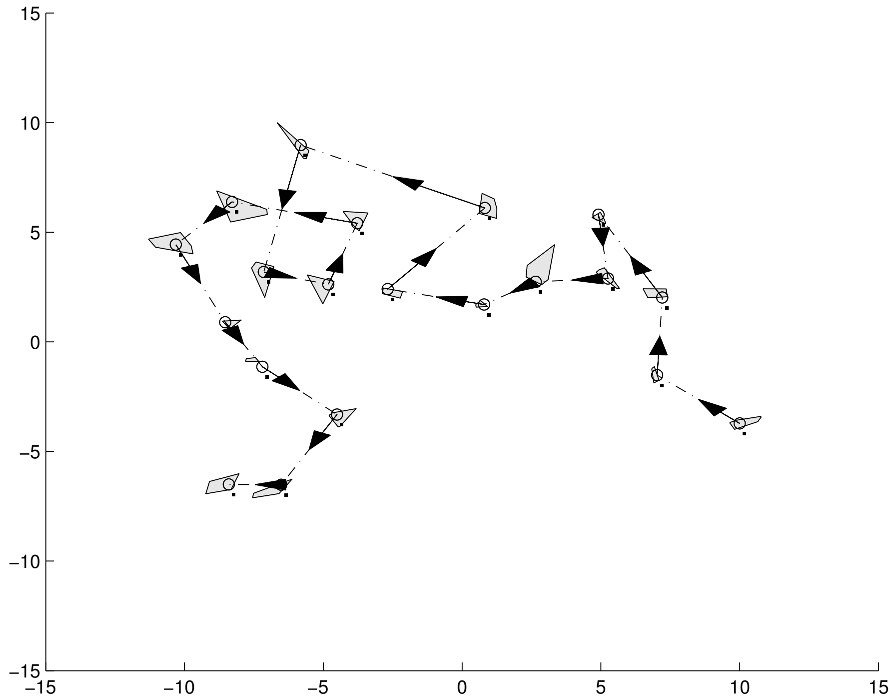

Figure 4.7: Algorithm employing the idea of a few violated constraints for small noise

The addition of a larger noise 0.5η resulted in the violation of too many constraints for the method of a few violated constraints to work. Thus instead (3.1) was applied, but still with 15 sensors and the threshold $c = 3\sqrt{2}$. Because of the large number of violated constraints the performance as shown in Figure 4.8 was predictably poor. With 40 sensors, however, the performance shown in Figure 4.9, was much better.

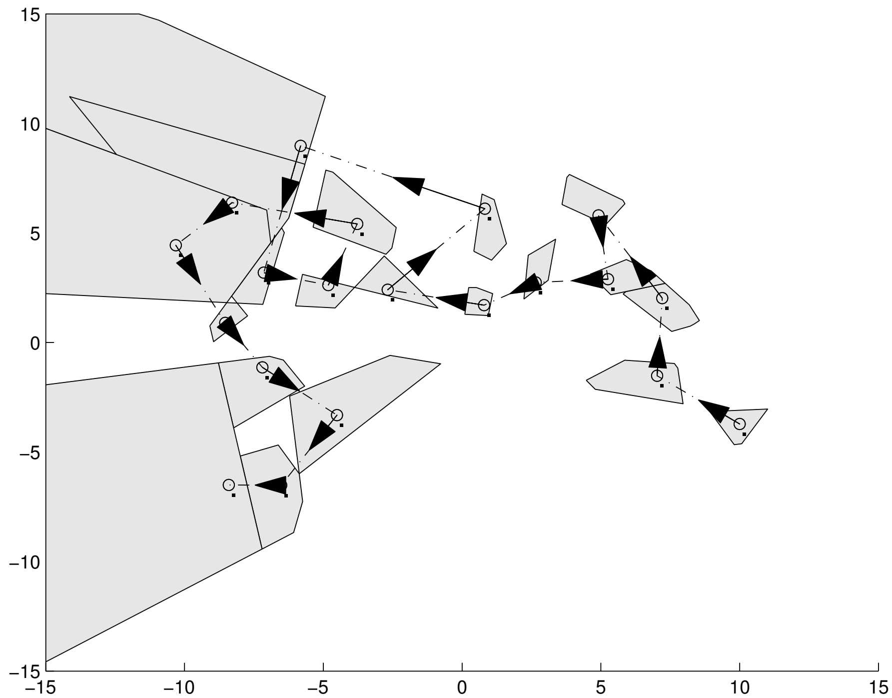

Figure 4.8: Large noise and small number of sensors

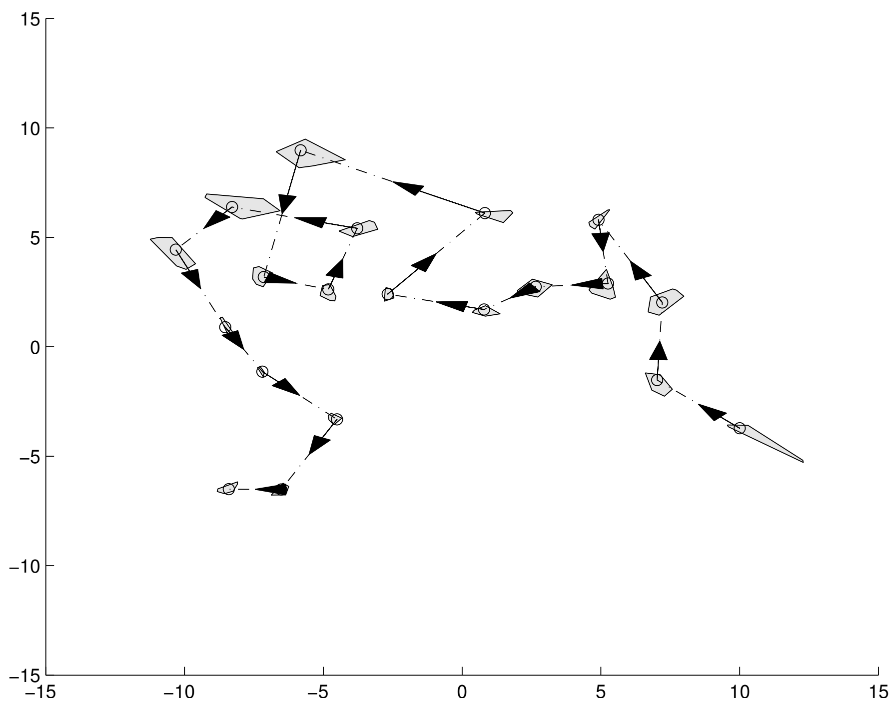

Figure 4.9: Large noise and large number of sensors

CHAPTER 5
CONCLUSION AND FUTURE WORK

5.1 Conclusion

Four algorithms for source localization are presented in this thesis that are particularly useful when the signal propagation model is not completely known. The only assumption was that the propagation model is monotonic with the distance between the source and sensors. Using this assumption we were able to come up with algorithms that could be applied when the signal strength model is unavailable. When choosing an algorithm, it is important to keep in mind that no single algorithm is perfect and applicable to any situation. One must take into consideration such factors as prior information on the unknown noise, shape of the source, number of measurements and others. For a practical application, often a combination of algorithms works best. For instance, even for a stationary source and stationary sensors, the robust algorithms based on the threshold developed in Chapter three for a moving source can be applied in addition to time averaging. One important feature of the algorithms we developed is also that it is applicable to many applications of source localization and tracking in which the propagation model is not known but monotonic with the distance of the source from the sensors.

5.2 Future work

Motivated from the work we have done, there are a number of natural extensions that can be made from the algorithms presented in the thesis. Of even great importance is the need to develop an algorithm using combined information of time averaging and thresholding. But an important future work is to develop an algorithm to deal with localizing and tracking multiple sources under the condition that there is no full knowledge of the propagation model of the source, assuming the propagation model is monotonic in distance of the sources from the sensor.

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